

Test Assignment #4

① Given $P = \$10,000$, $r = 2.5\% = 0.025$ and $n = 12$

(i) when $t = 2$ years,

$$F(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\Rightarrow F(2) = 10000 \left(1 + \frac{0.025}{12}\right)^{(12)(2)} \approx \$10,512.16$$

(ii) We want $F(t) \geq \$13,000$

$$\Rightarrow P\left(1 + \frac{r}{n}\right)^{nt} \geq \$13,000$$

$$\Rightarrow 10000 \left(1 + \frac{0.025}{12}\right)^{12t} \geq 13000$$

$$\Rightarrow \left(1 + \frac{0.025}{12}\right)^{12t} \geq \frac{13000}{10000}$$

$$\Rightarrow \left(1 + \frac{0.025}{12}\right)^{12t} \geq 1.3$$

$$\Rightarrow 12t \geq \log_{\left(1 + \frac{0.025}{12}\right)} 1.3$$

$$\Rightarrow t \geq \frac{1}{12} \log_{\left(1 + \frac{0.025}{12}\right)} 1.3$$

$$\Rightarrow t \geq \frac{1}{12} \cdot \frac{\log_{10} 1.3}{\log_{10} \left(1 + \frac{0.025}{12}\right)} \quad \text{OR} \quad t \geq \frac{1}{12} \frac{\ln 1.3}{\ln \left(1 + \frac{0.025}{12}\right)}$$

$$\Rightarrow t \geq 10.505499 \text{ years}$$

$$\Rightarrow t \geq 10 \text{ years } 6.06 \text{ months}$$

$\therefore$ It will take 10 years 7 months for the investment to grow to at least \$13,000.

$$(iii) r_{\text{eff}} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$\Rightarrow r_{\text{eff}} = \left(1 + \frac{0.025}{12}\right)^{12} - 1 \approx 0.025288$$

$$\approx 2.53\%$$

(2) Given $P = \$250,000$

and $r_{\text{eff}} = 3\% = 0.03$

$$(i) \quad F(t) = Pe^{rt}$$

We know that $r_{\text{eff}} = e^r - 1$

$$\Rightarrow e^r = r_{\text{eff}} + 1$$

$$\Rightarrow e^r = 0.03 + 1 = 1.03$$

At $t = 3$ years,

$$F(3) = (250,000)(1.03)^3 = \$273,181.75$$

$$(ii) \quad F(t) = Pe^{rt}$$

We want $F(t) = \$300,000$

$$\text{so } 300000 = (250000)(1.03)^t$$

$$\Rightarrow (1.03)^t = \frac{300000}{250000} = \frac{6}{5}$$

$$\Rightarrow t = \log_{1.03} \frac{6}{5}$$

$$\Rightarrow t = \frac{\log_{10} \frac{6}{5}}{\log_{10} 1.03}$$

$$\text{OR } t = \frac{\ln \frac{6}{5}}{\ln 1.03}$$

$$\Rightarrow t = 6.17 \text{ years}$$

$$(iii) \quad e^r = 1.03$$

$$\Rightarrow r = \ln 1.03 \approx 0.029559$$

$$\approx 2.96\%$$

③ Given $s(0) = 50$, $s(30) = 250$
and $\lim_{t \rightarrow \infty} s(t) = 1000$

(i) Learning Curve Model

$$s(t) = L - ae^{-kt}$$

When $t=0$, $s(0) = L - a = 50$

and $\lim_{t \rightarrow \infty} s(t) = L = 1000$

So $1000 - a = 50$

$\Rightarrow a = 1000 - 50 = 950$

And when $t=30$, $s(30) = 250$

So $250 = 1000 - 950e^{-k(30)}$

$\Rightarrow 950e^{-30k} = 1000 - 250 = 750$

$\Rightarrow e^{-30k} = \frac{750}{950} = \frac{15}{19}$

$\Rightarrow -30k = \log_e \frac{15}{19} = \ln \frac{15}{19}$

$\Rightarrow k = \frac{\ln \frac{15}{19}}{-30} \approx 0.00787963$

$\therefore s(t) = 1000 - 950e^{-0.00787963t}$

(ii) On the 51st day, we have $t=50$

$$s(50) = 1000 - 950e^{-0.00787963(50)}$$

$\approx 359.35 \approx 359$ customers

(iii) When $s(t) = 500$,

$$500 = 1000 - 950e^{-0.00787963t}$$

$\Rightarrow 950e^{-0.00787963t} = 1000 - 500 = 500$

$\Rightarrow e^{-0.00787963t} = \frac{500}{950} = \frac{10}{19}$

$\Rightarrow -0.00787963t = \ln \frac{10}{19}$

$\Rightarrow t = \frac{\ln \frac{10}{19}}{-0.00787963} \approx 81.46$

$\therefore$ 500 customers on the 82nd day.

(4)

Given

$$y = \frac{(x^2+2)^3(x+3)^2}{(x^3-2)^4}$$

$$\Rightarrow \ln y = \ln \frac{(x^2+2)^3(x+3)^2}{(x^3-2)^4}$$

$$= \ln(x^2+2)^3 + \ln(x+3)^2 - \ln(x^3-2)^4$$

$$\Rightarrow \ln y = 3 \ln(x^2+2) + 2 \ln(x+3) - 4 \ln(x^3-2)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = 3 \frac{2x}{x^2+2} + 2 \frac{1}{x+3} - 4 \frac{3x^2}{x^3-2}$$

$$\Rightarrow \frac{dy}{dx} = y \left(\frac{6x}{x^2+2} + \frac{2}{x+3} - \frac{12x^2}{x^3-2} \right)$$